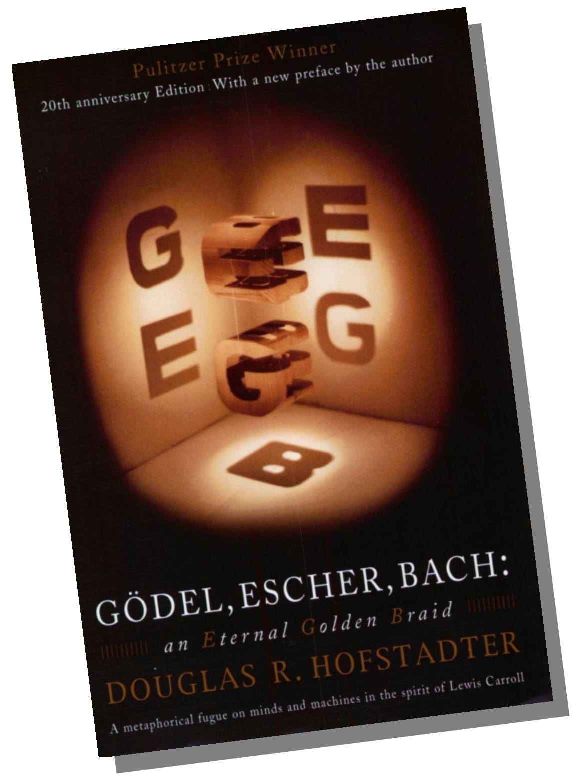
Constructability of Trip-lets

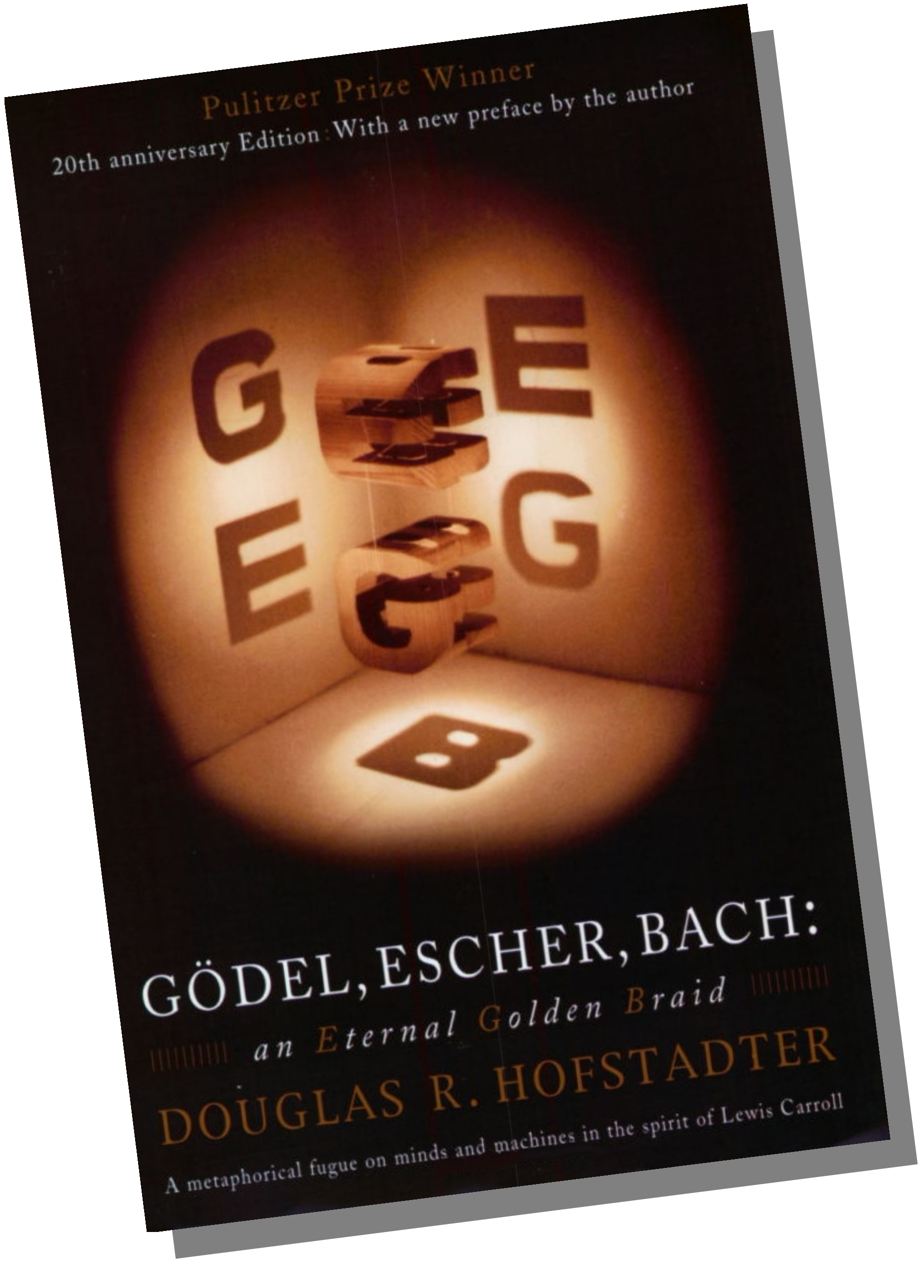


Jeroen Keiren   Freek van Walderveen*   Alexander Wolff

Eindhoven University of Technology, the Netherlands

*Now at **MADALGO, University of Aarhus, Denmark**

Trip-lets



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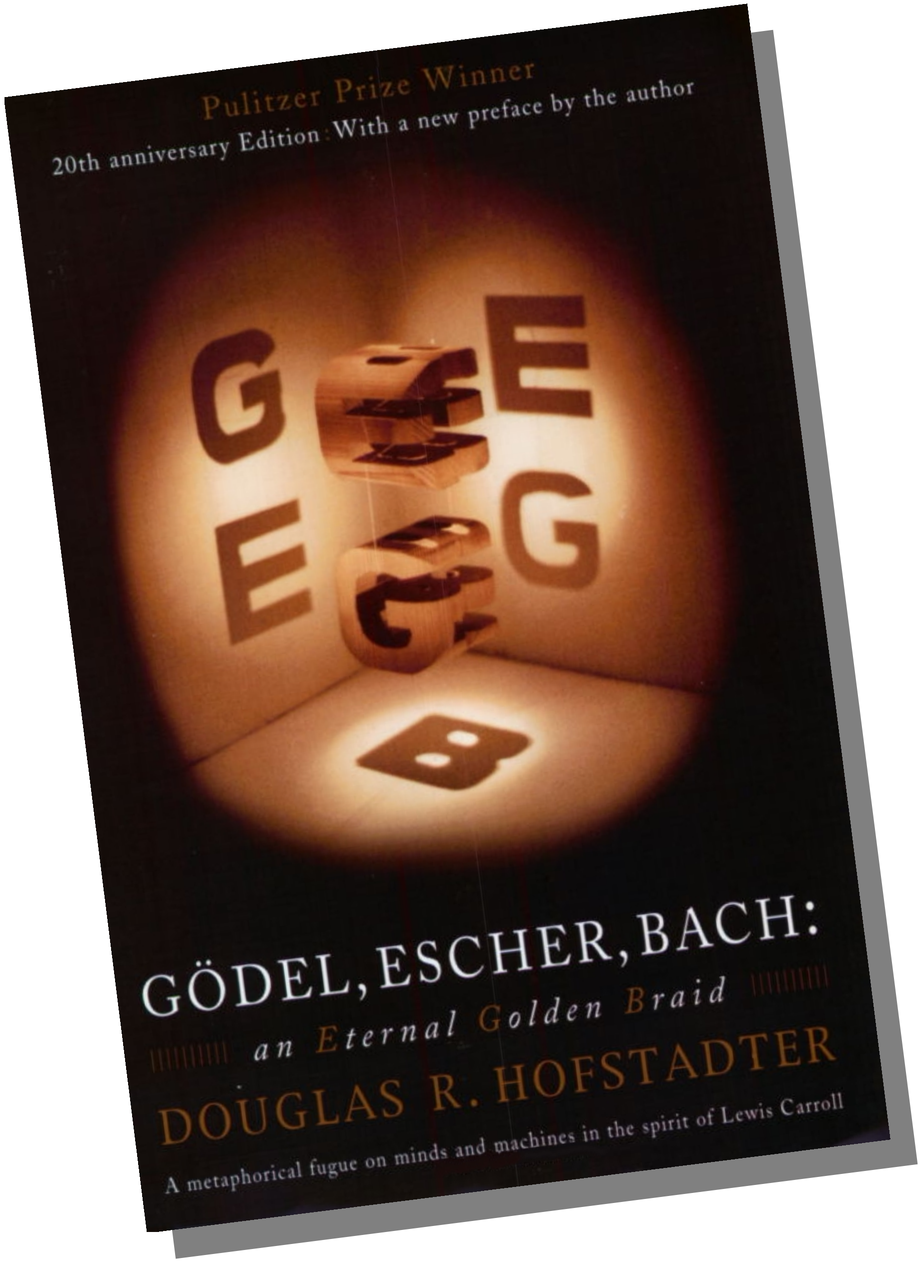

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Conversely:
construction for three given letters (**shapes**) by removing from a cube all material obstructing the shadows’ negatives.

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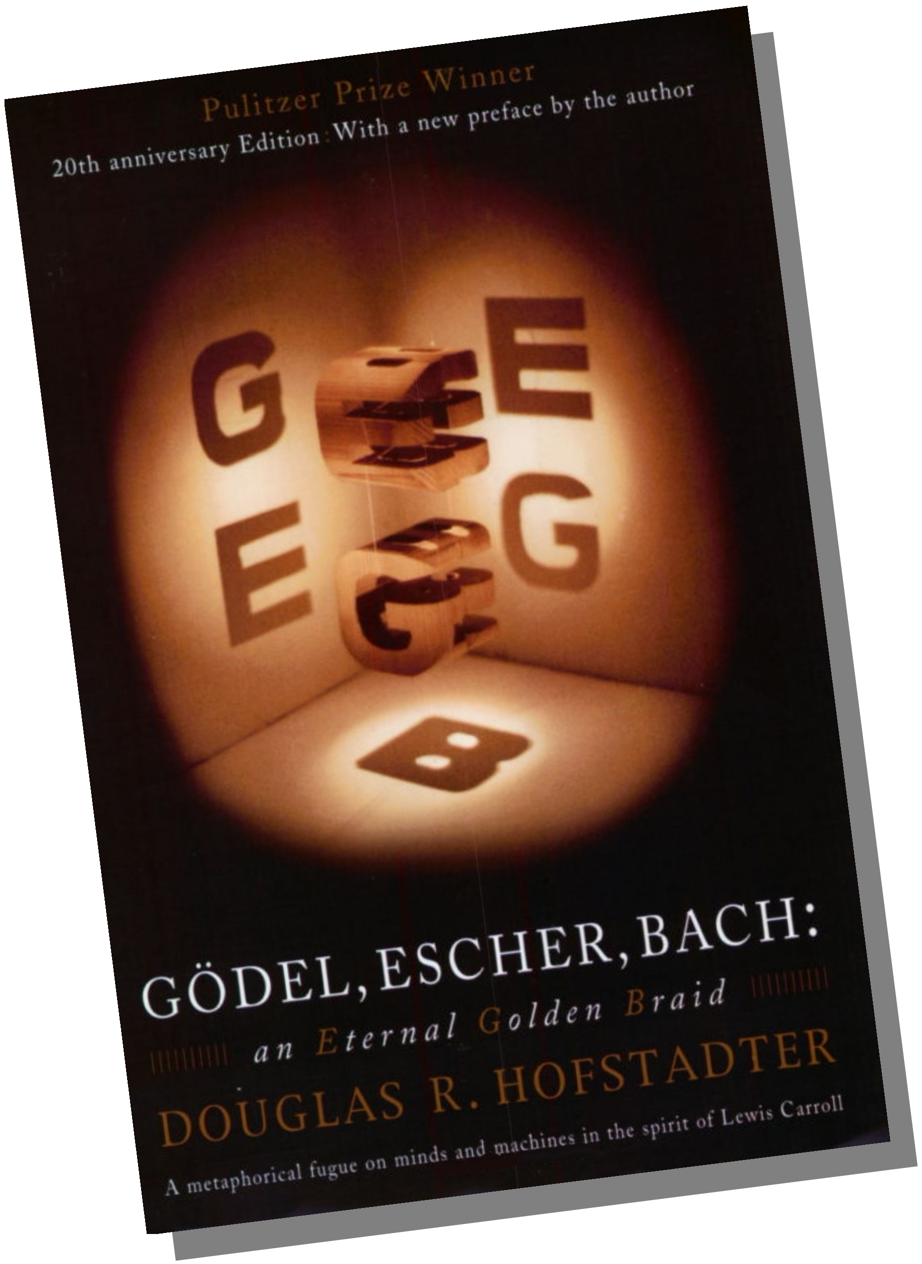

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Is this always possible?

Or are there combinations of letters for which it does not work out?

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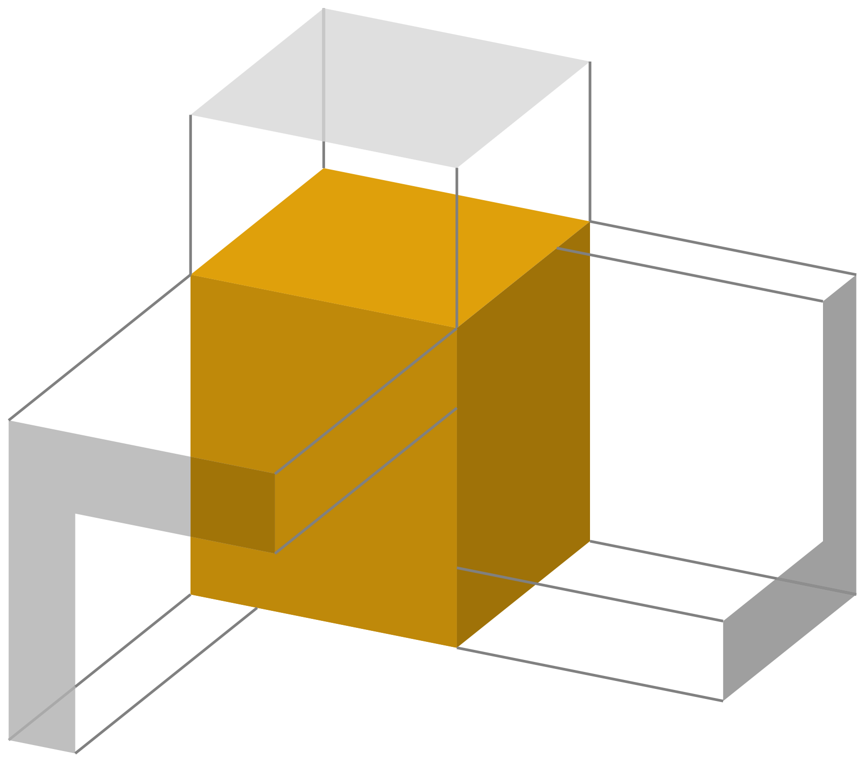

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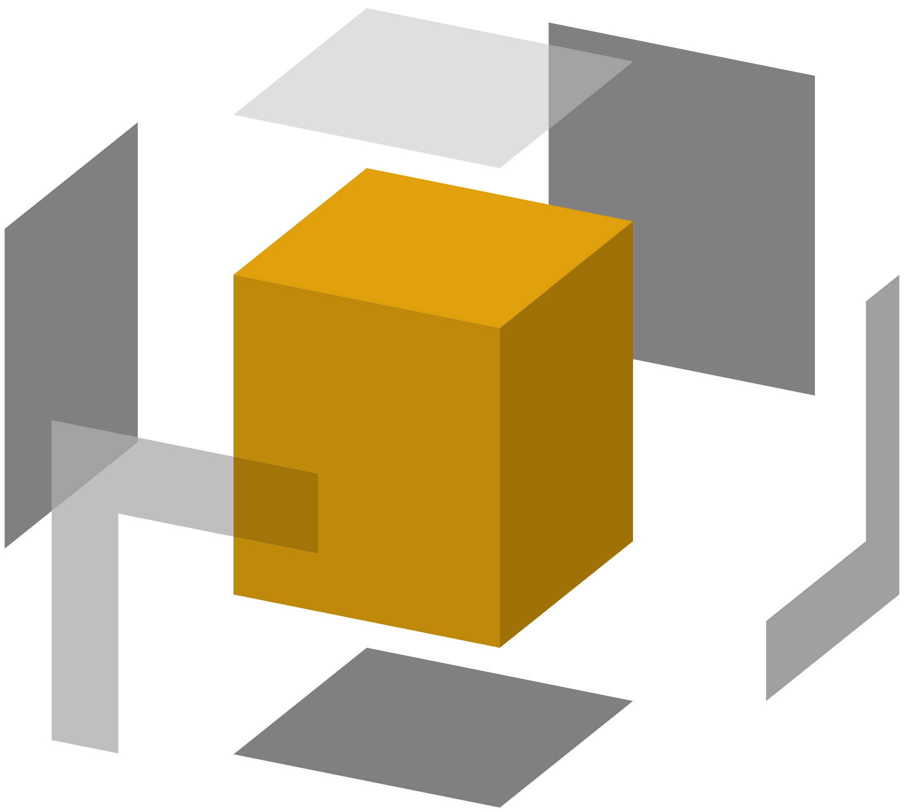

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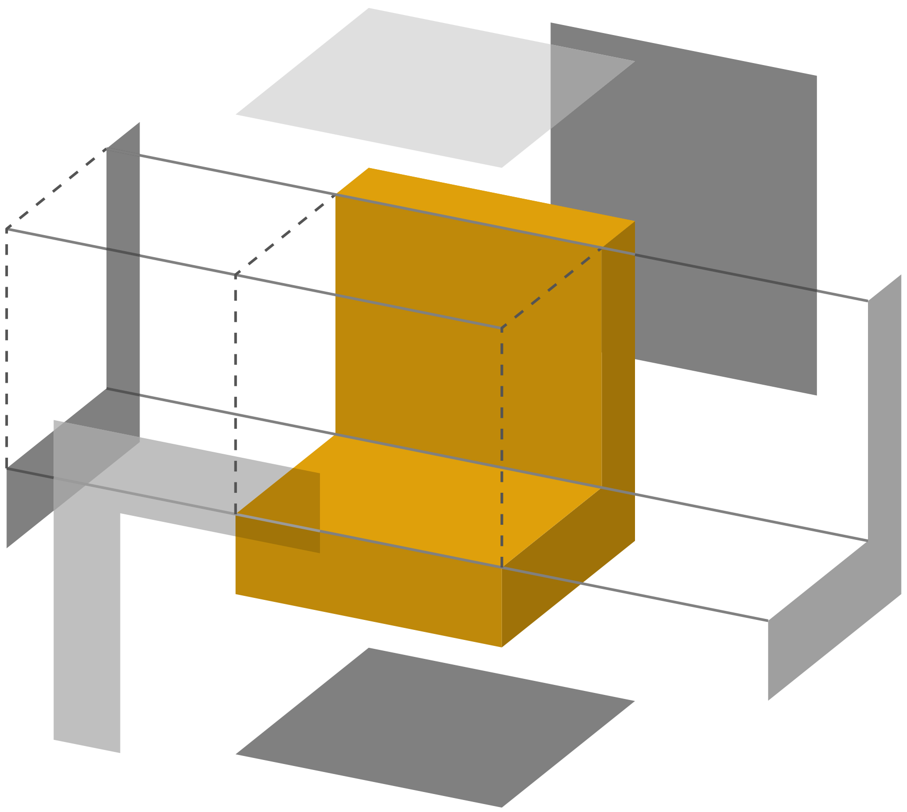

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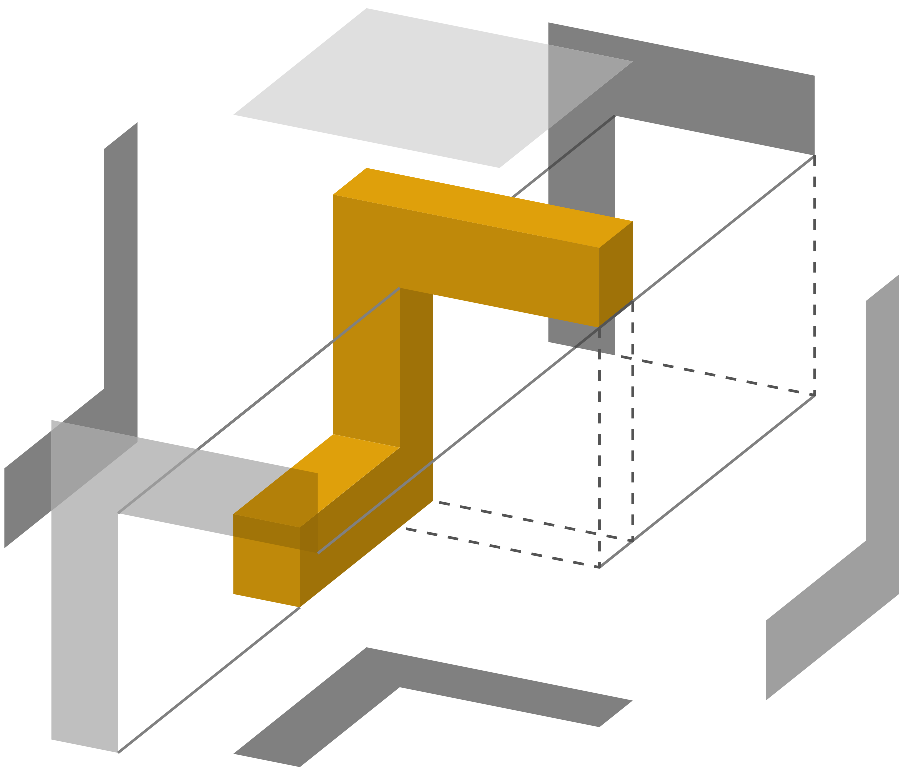

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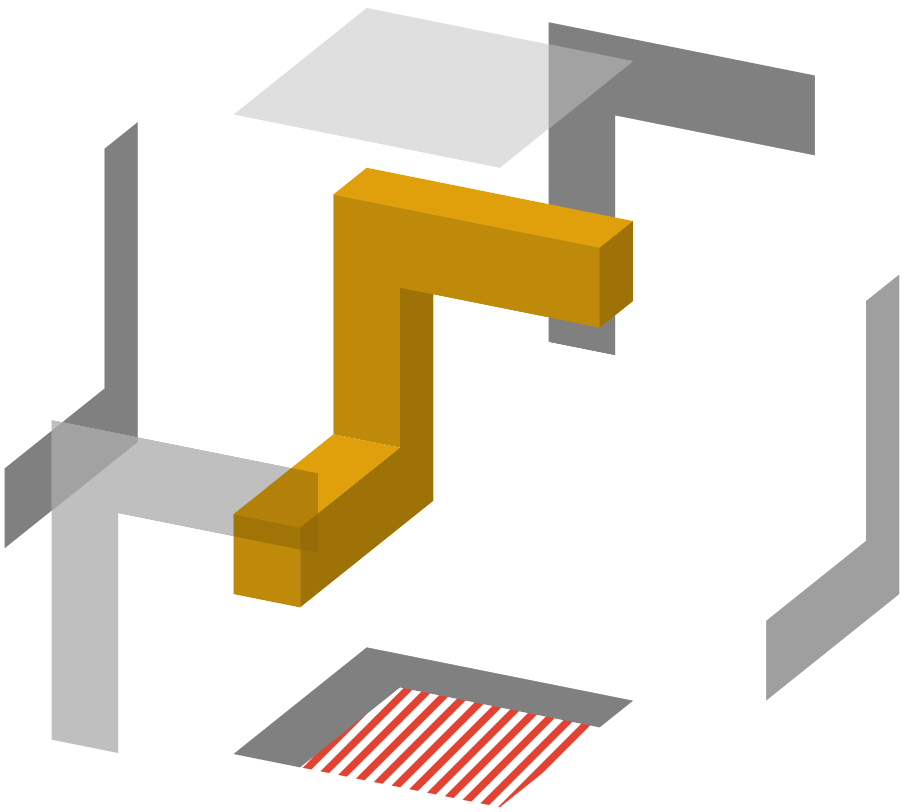

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Is this always possible? **No!**

Or are there combinations of letters for which it does not work out?

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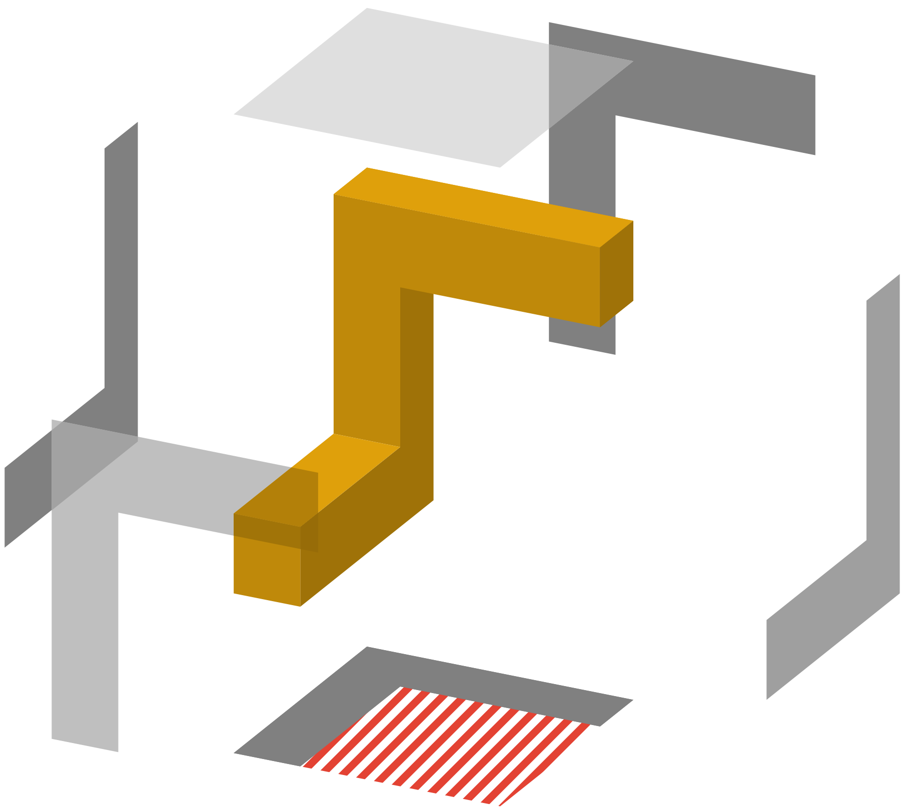

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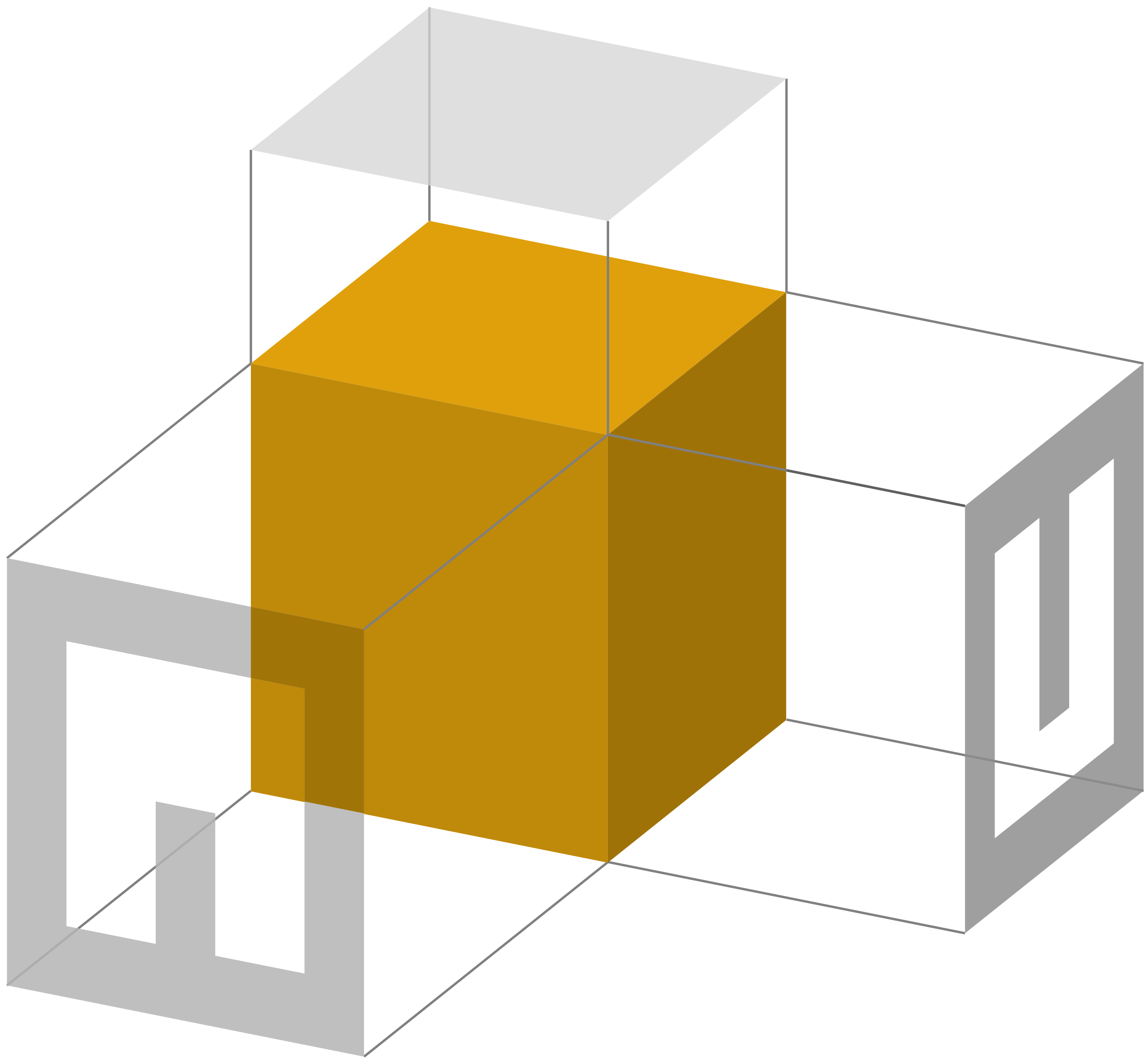
Question 1:
For which shapes is this possible?

Trip-lets

a second example



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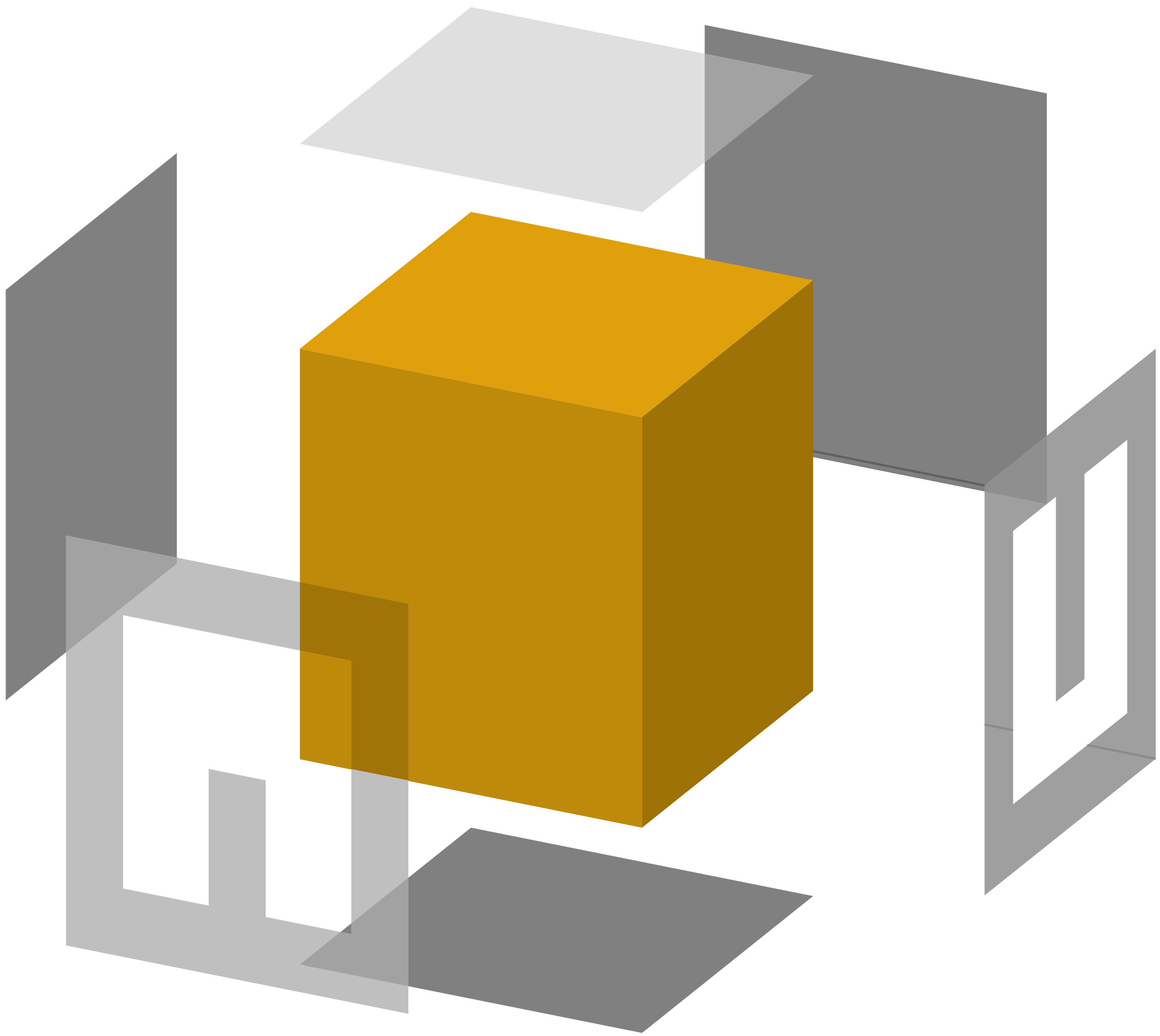

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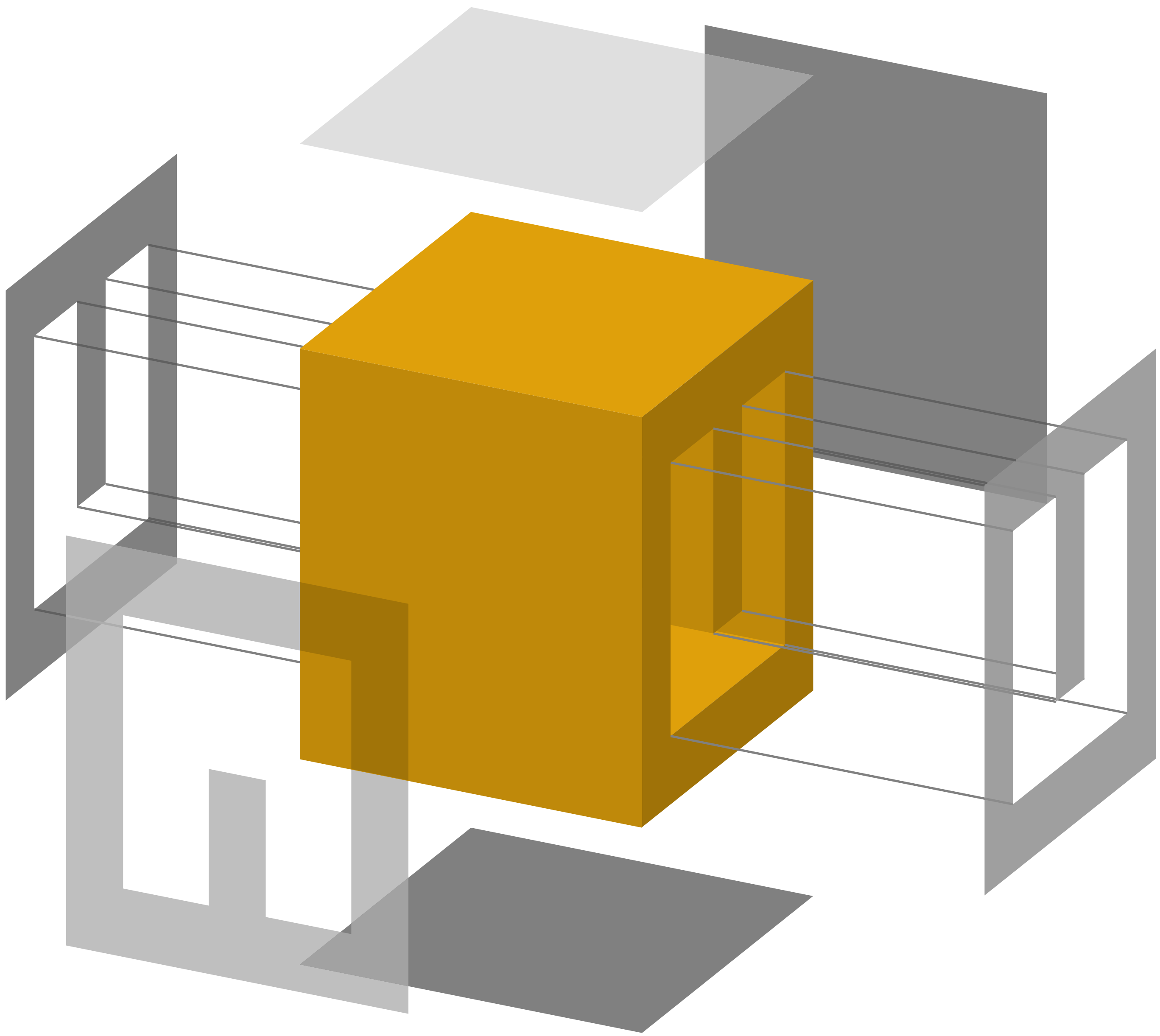

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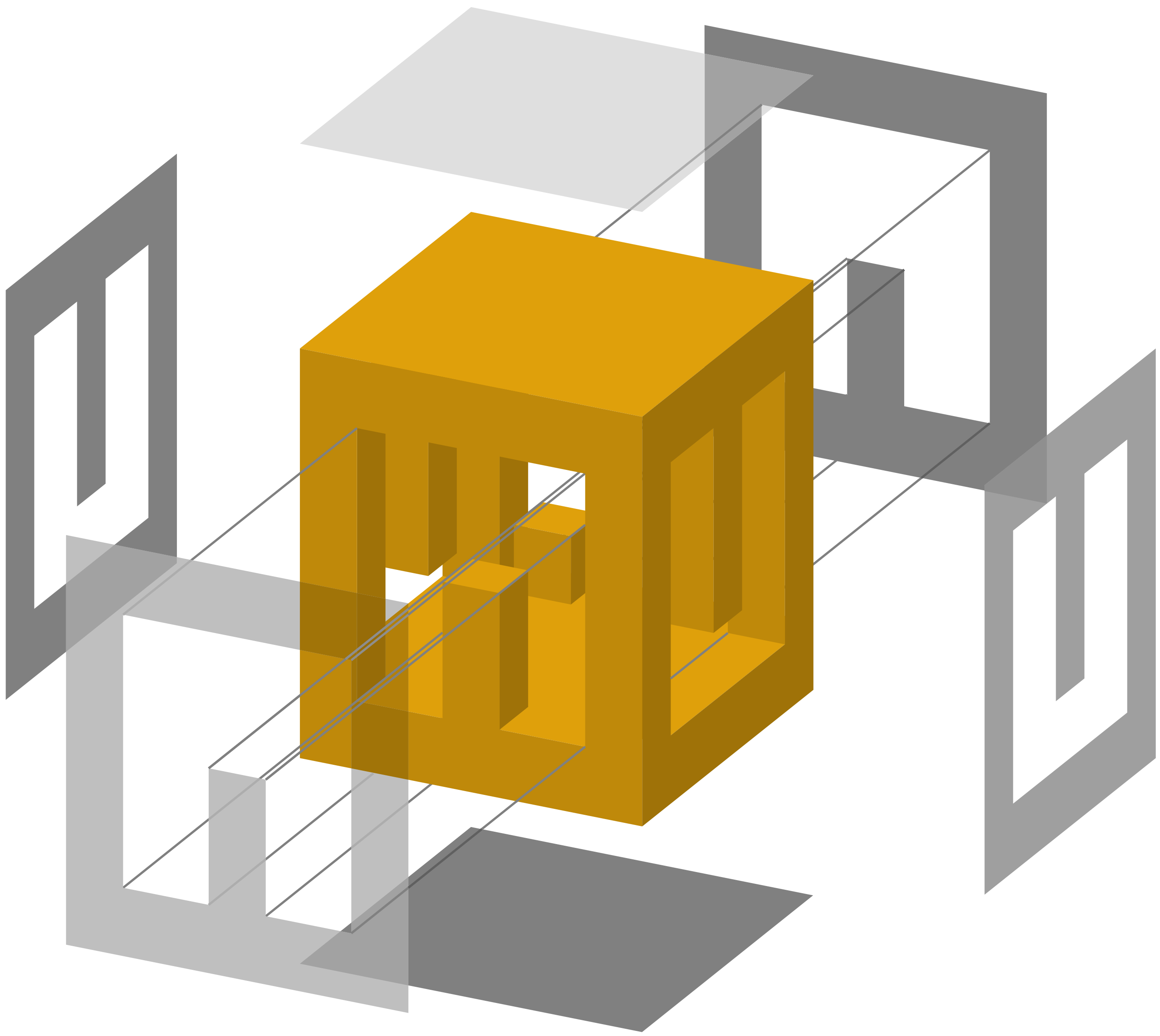

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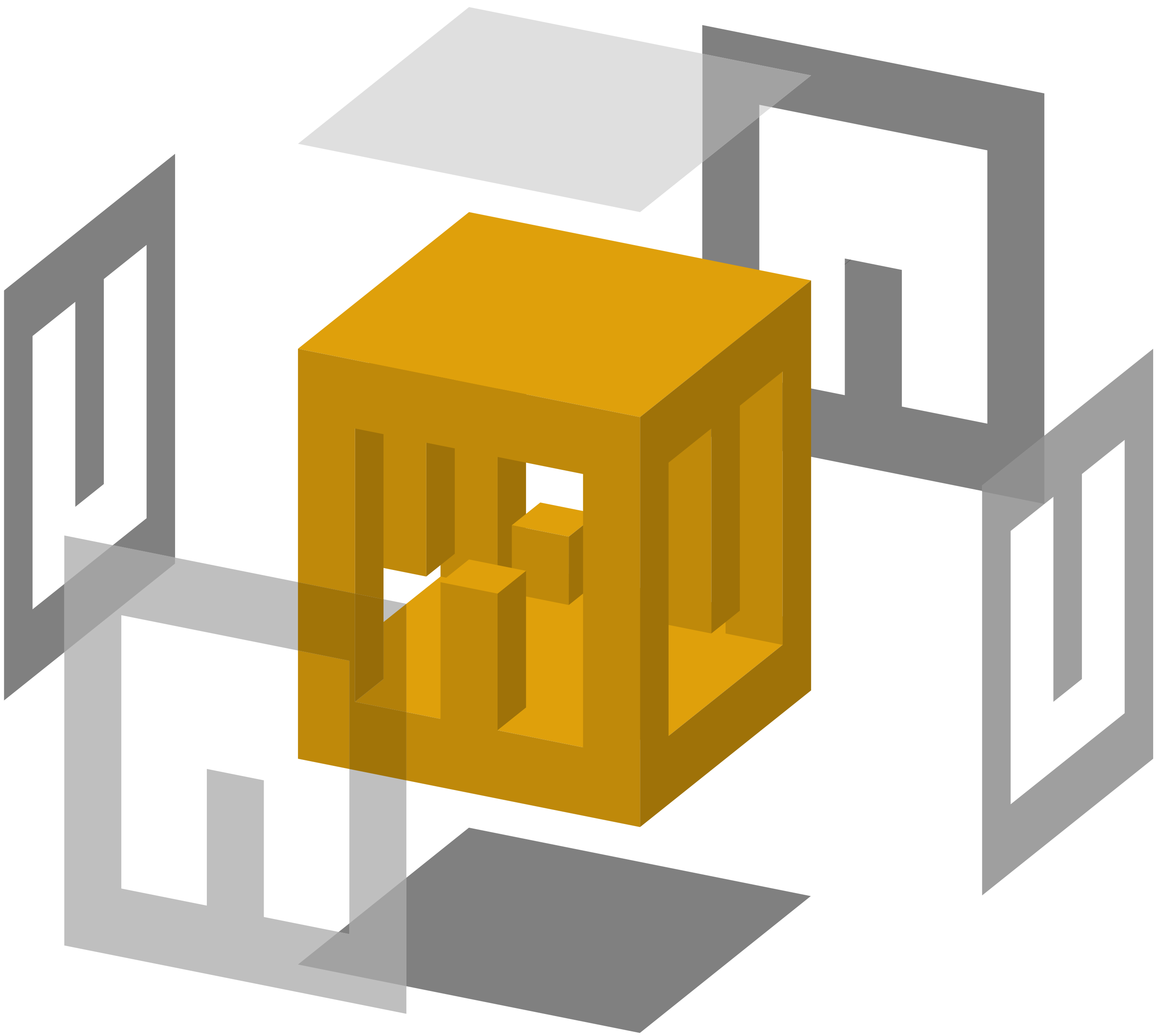

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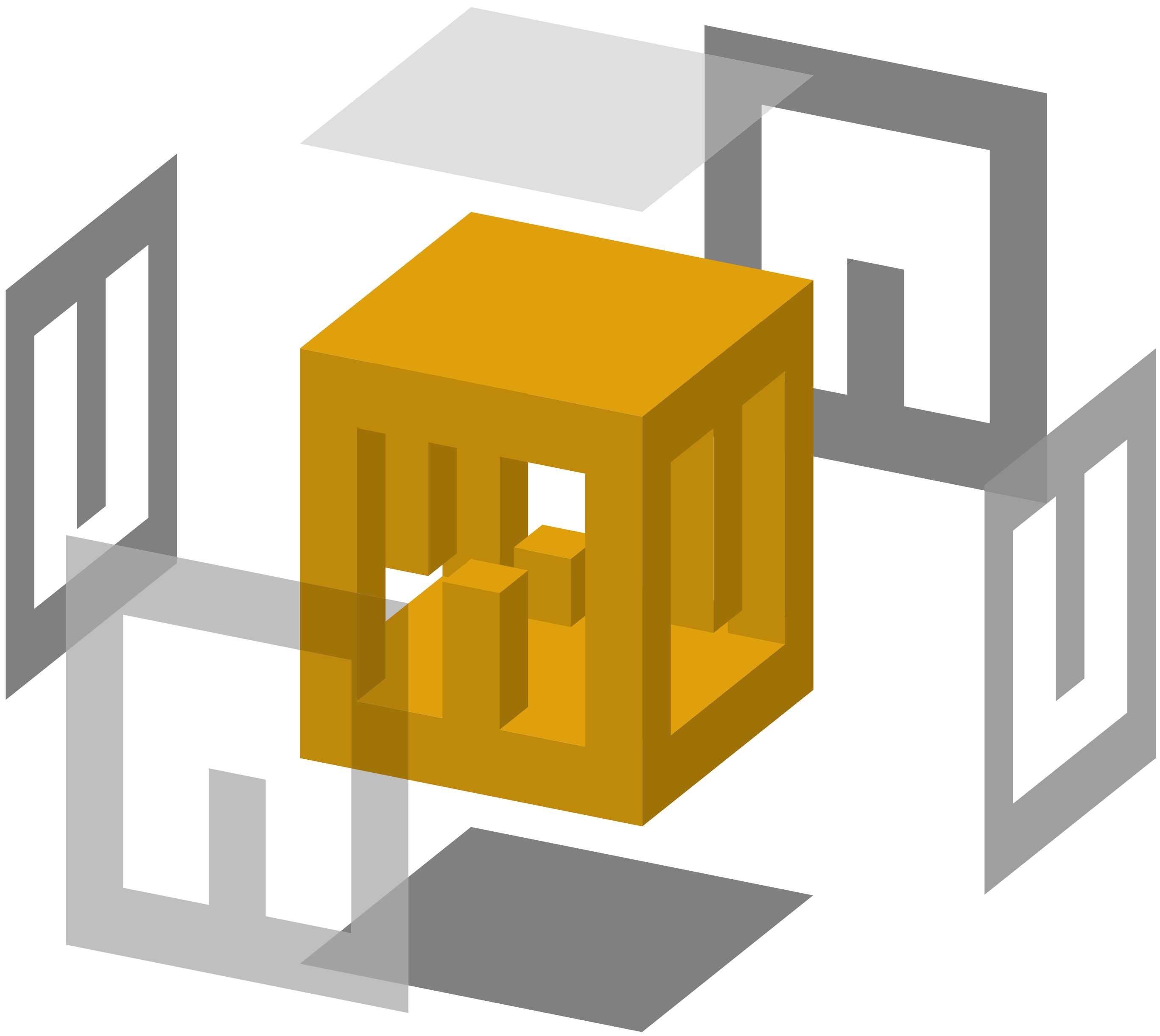

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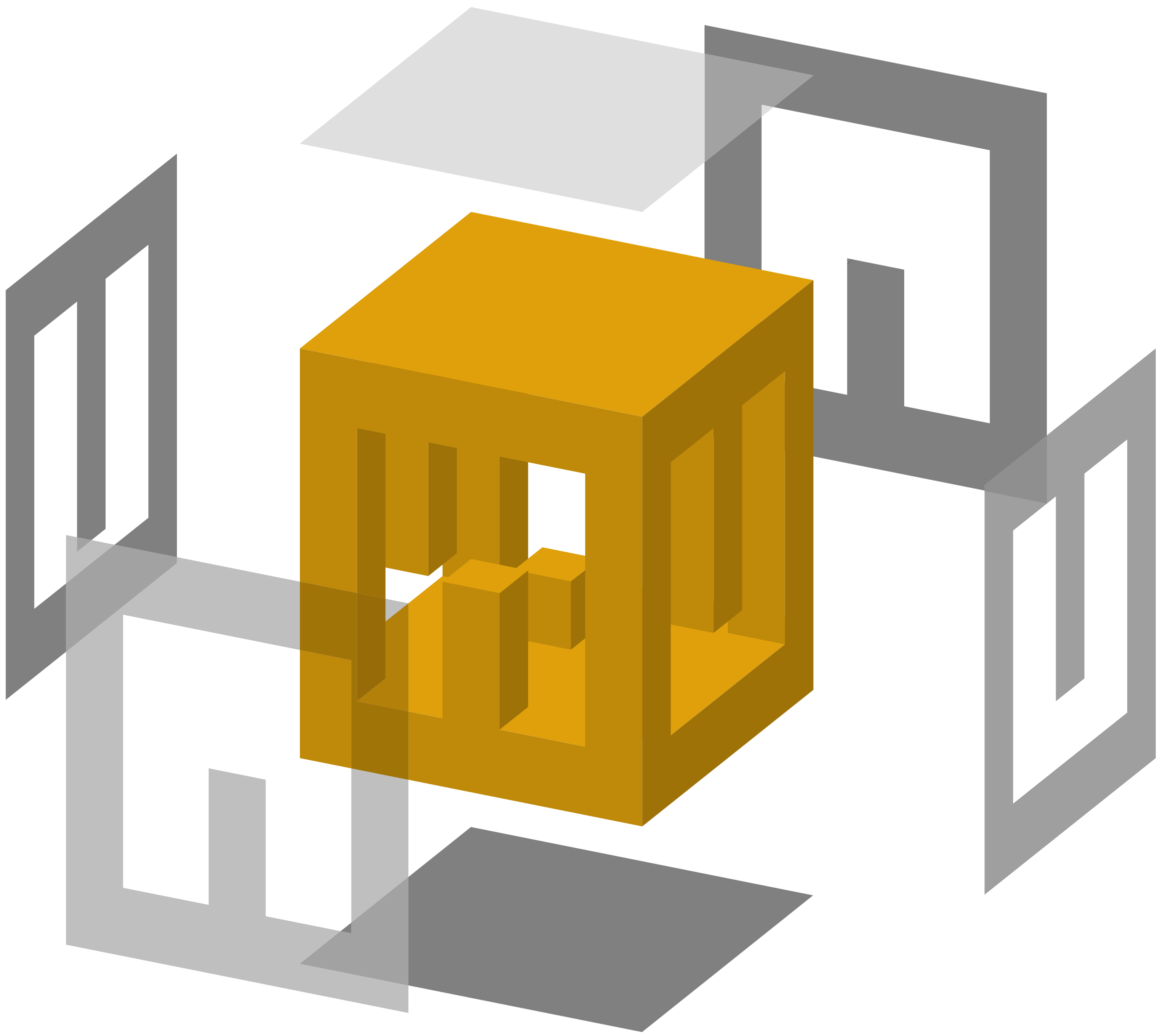

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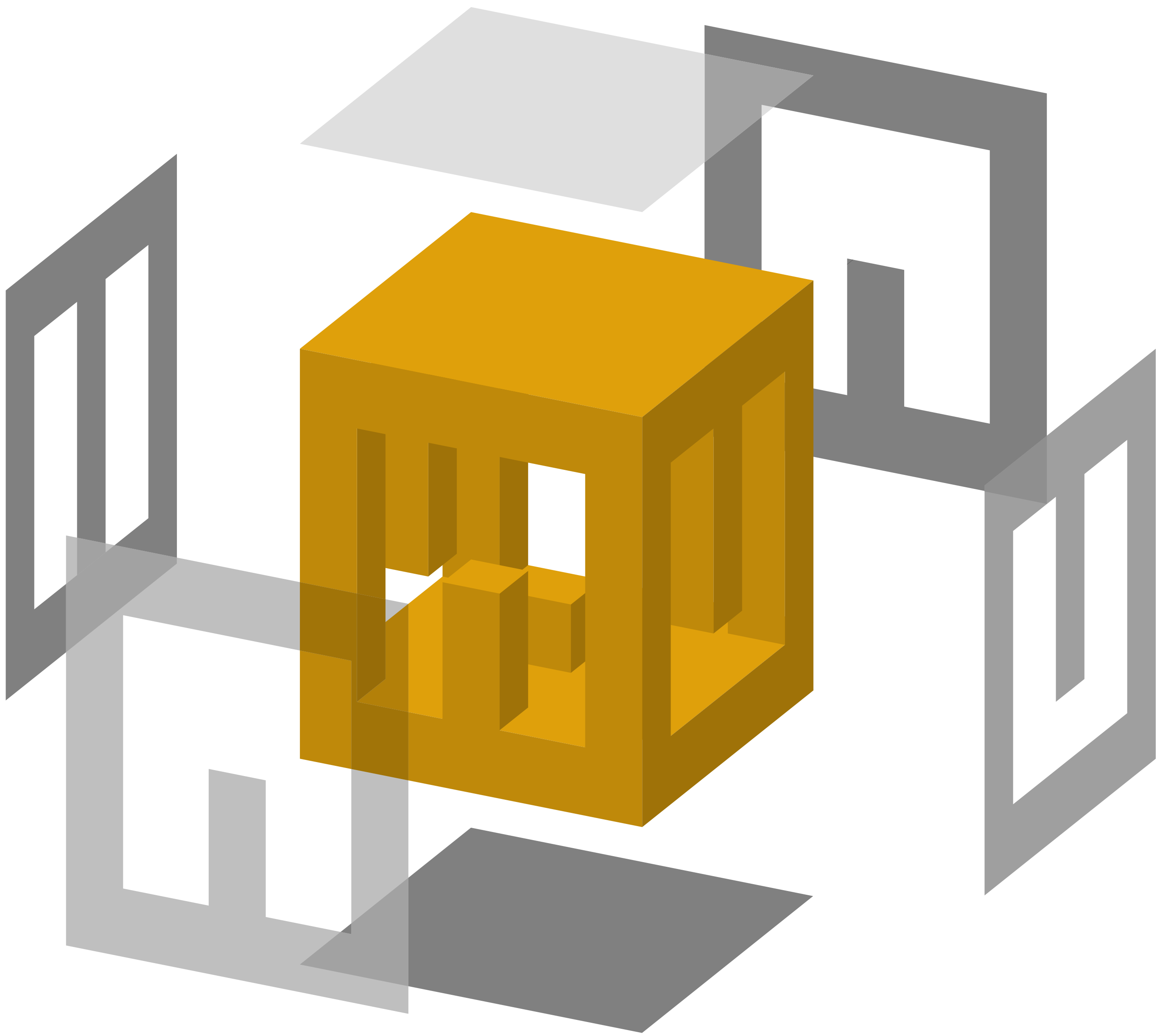

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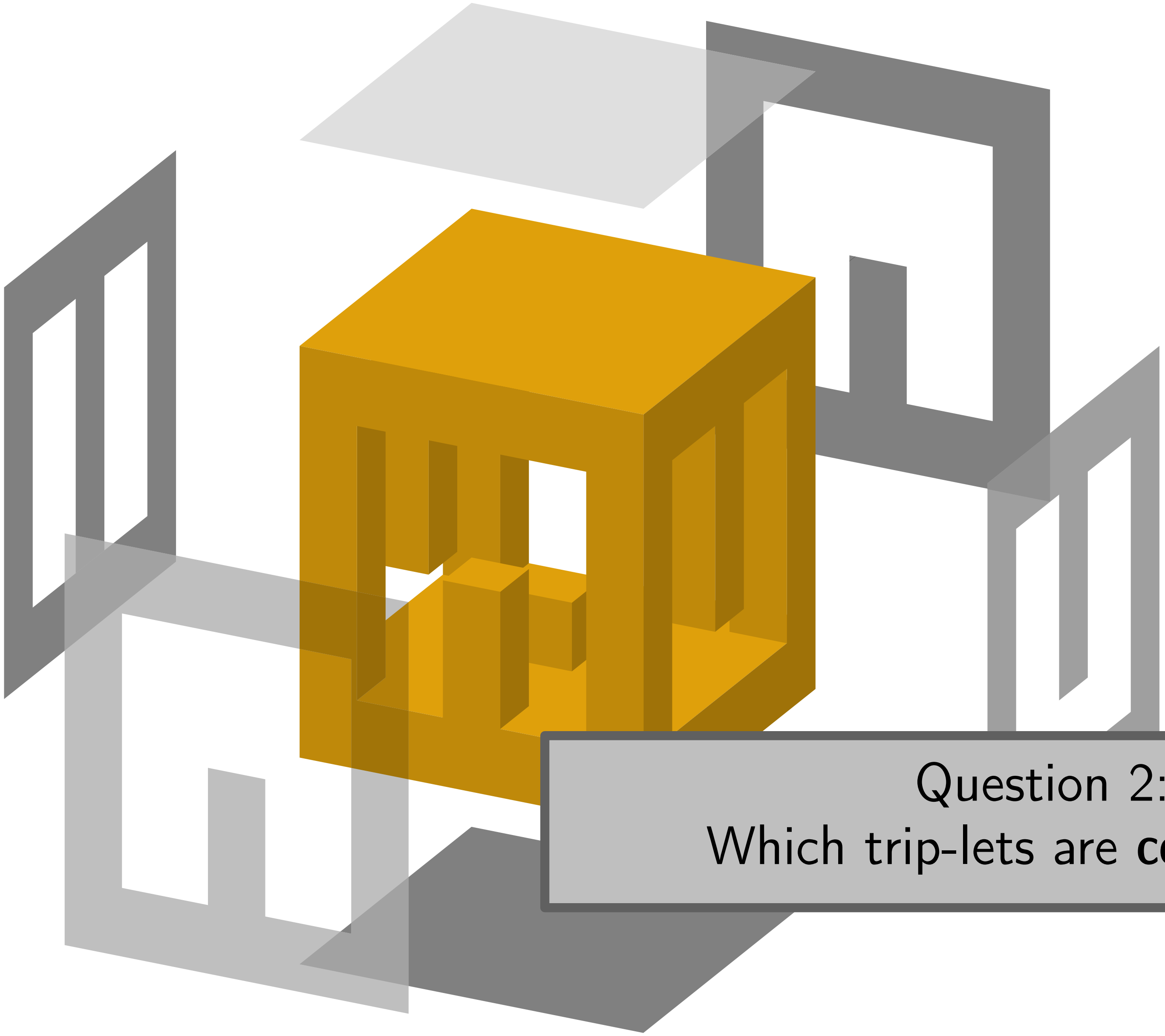

Question 2:
Which trip-lets are **connected**?

Overview

Given three polygonal shapes with n vertices in total.

- How many vertices can a trip-let have?
- How many vertices can its shadows have?
- How fast can we find out whether a trip-let can be made? (Q1)
- How fast can we find out whether the trip-let is connected? (Q2)

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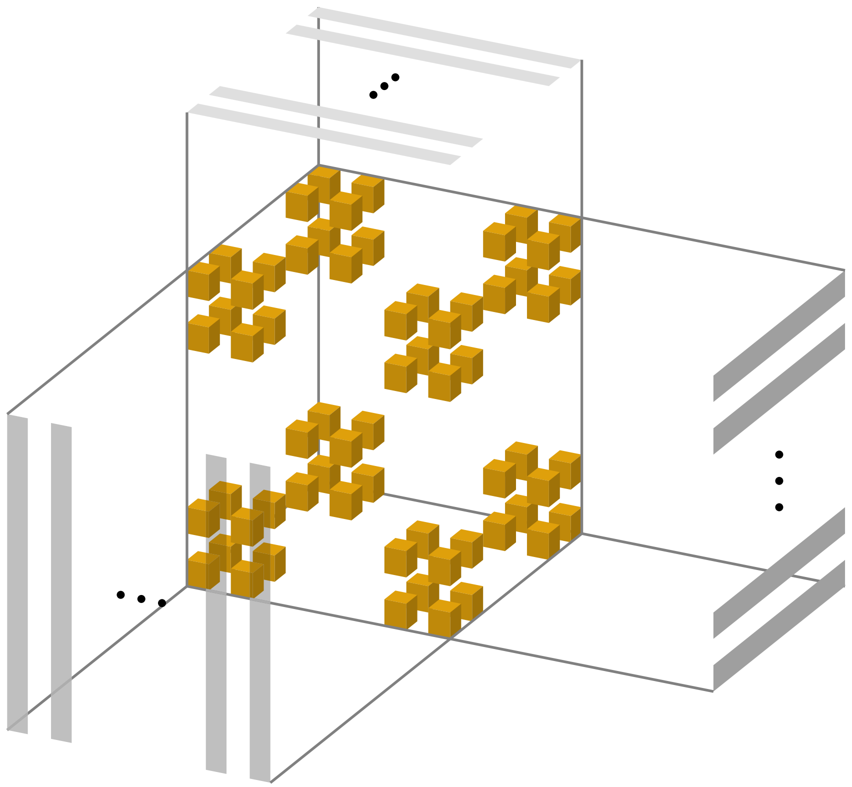

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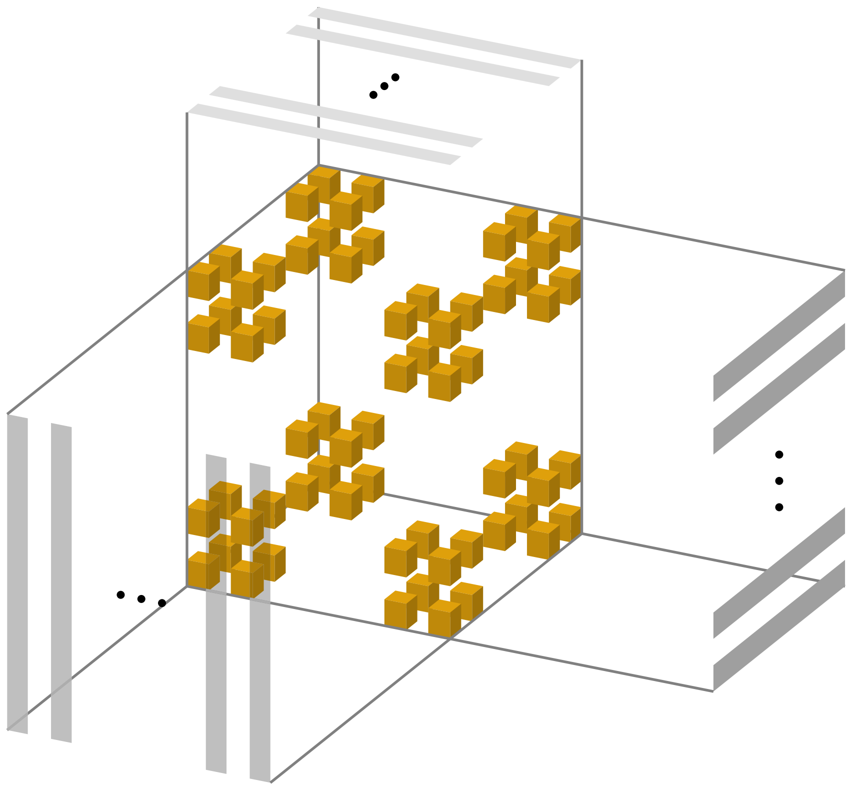

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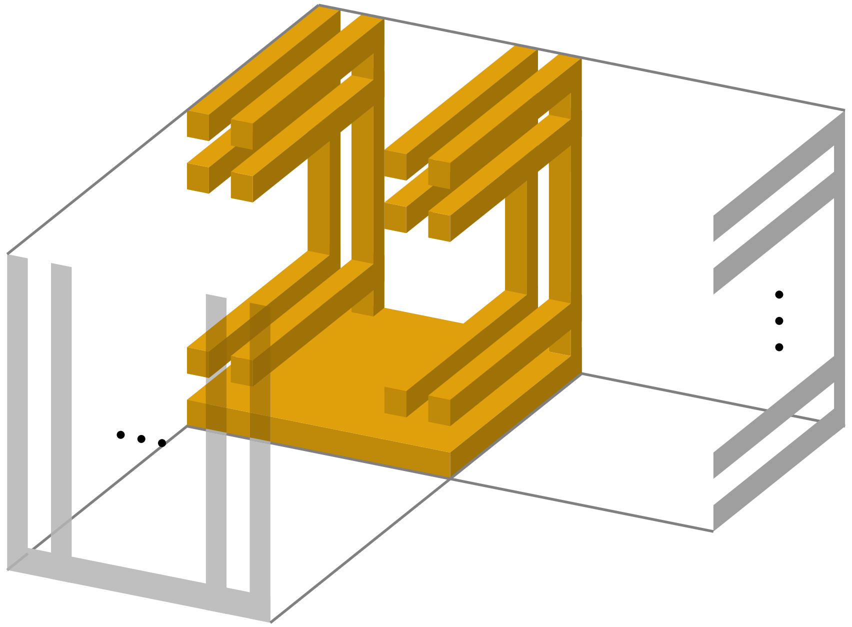

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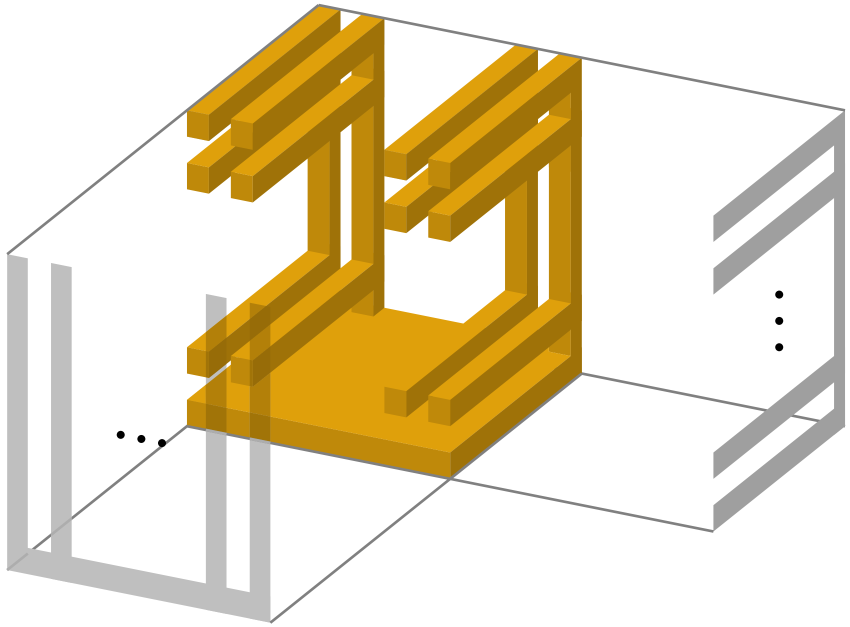
- but can it get any worse?
  (we don't think so)



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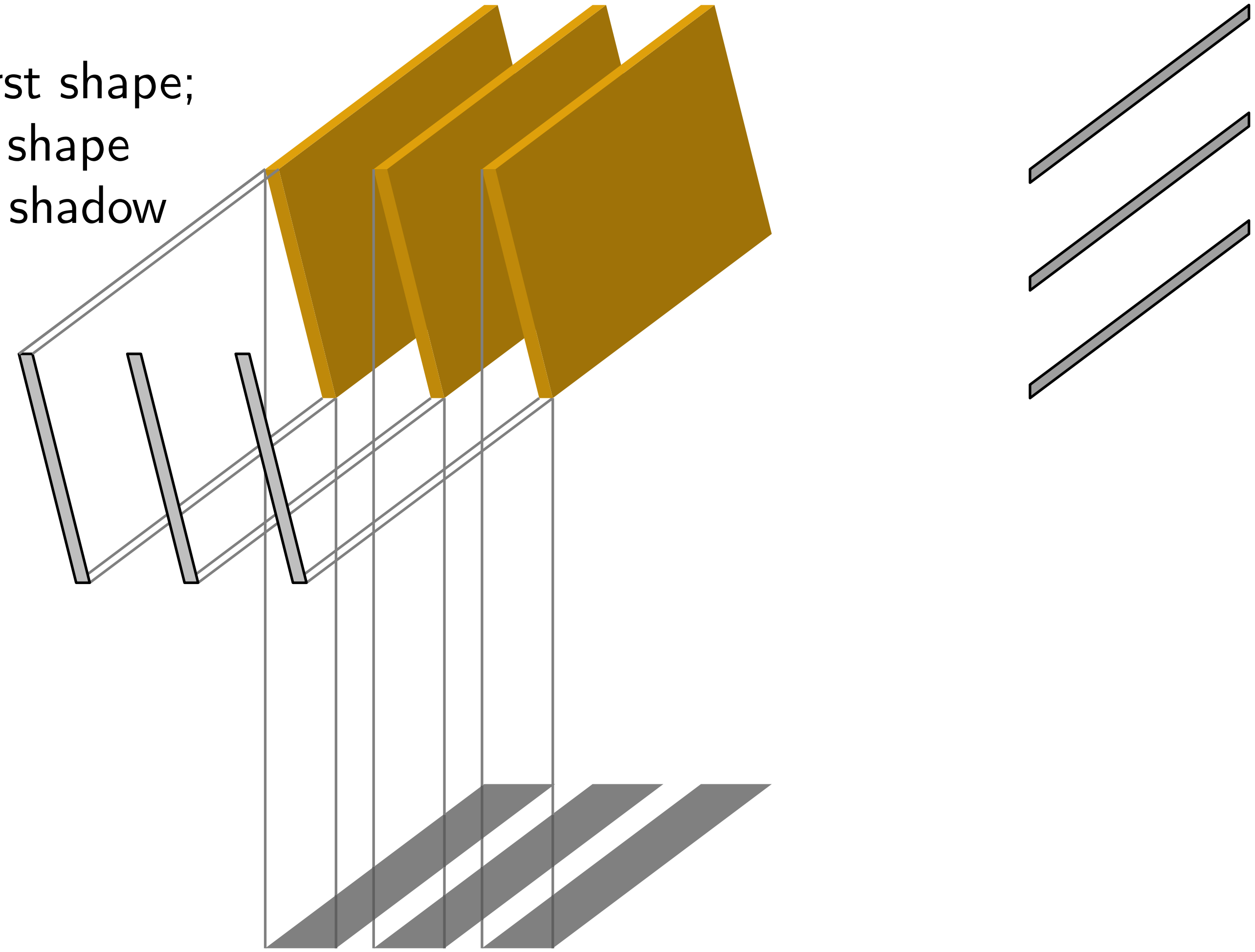

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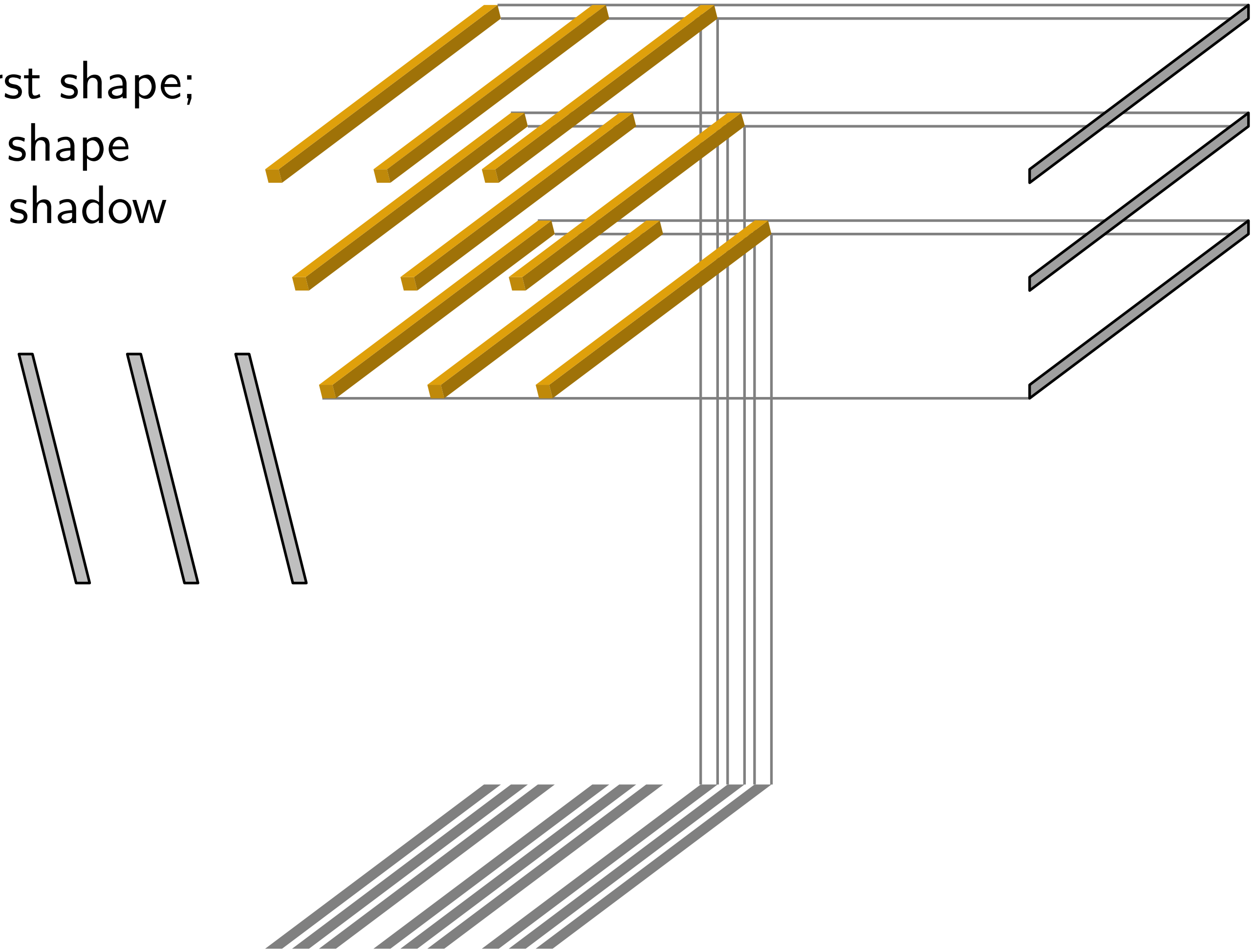

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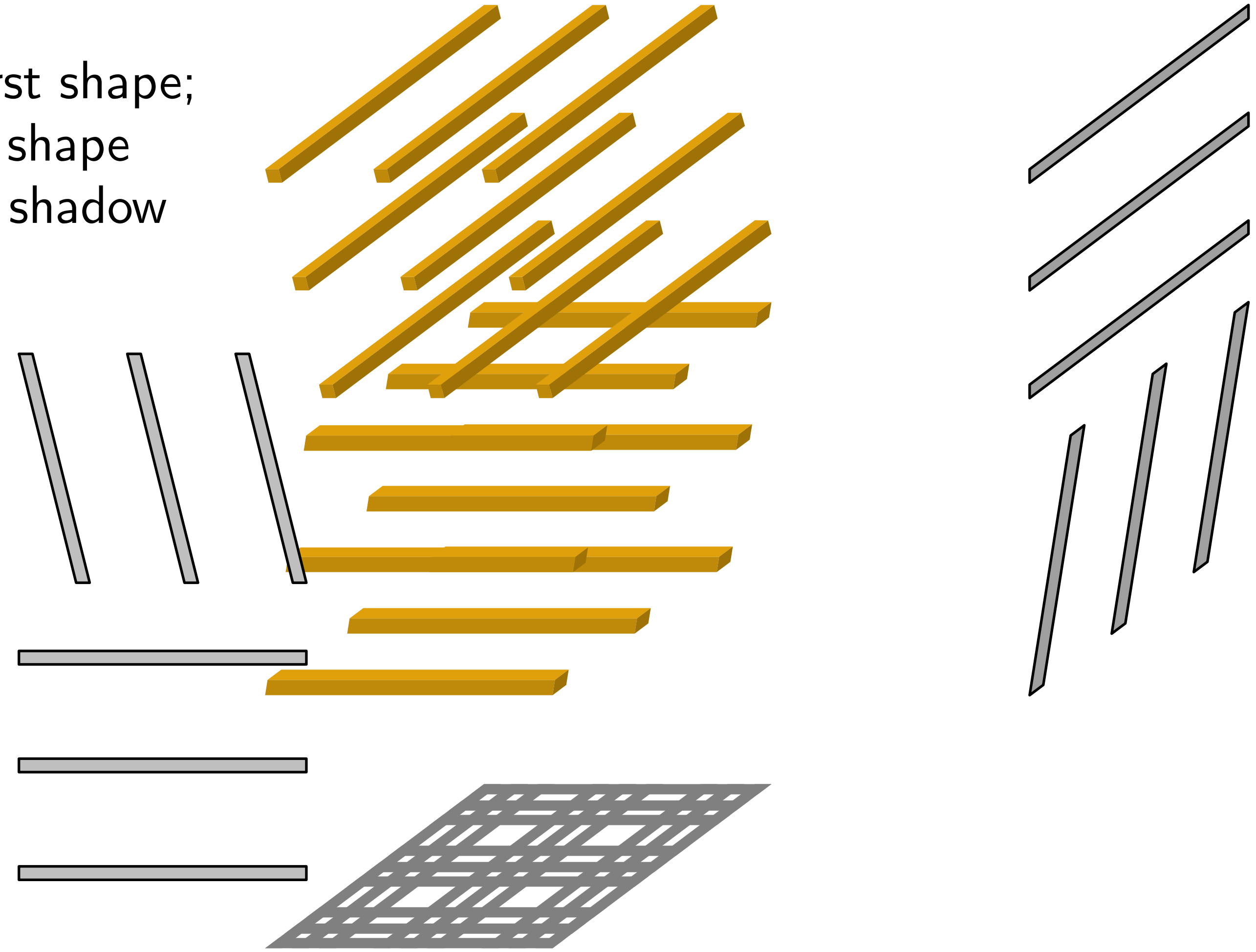

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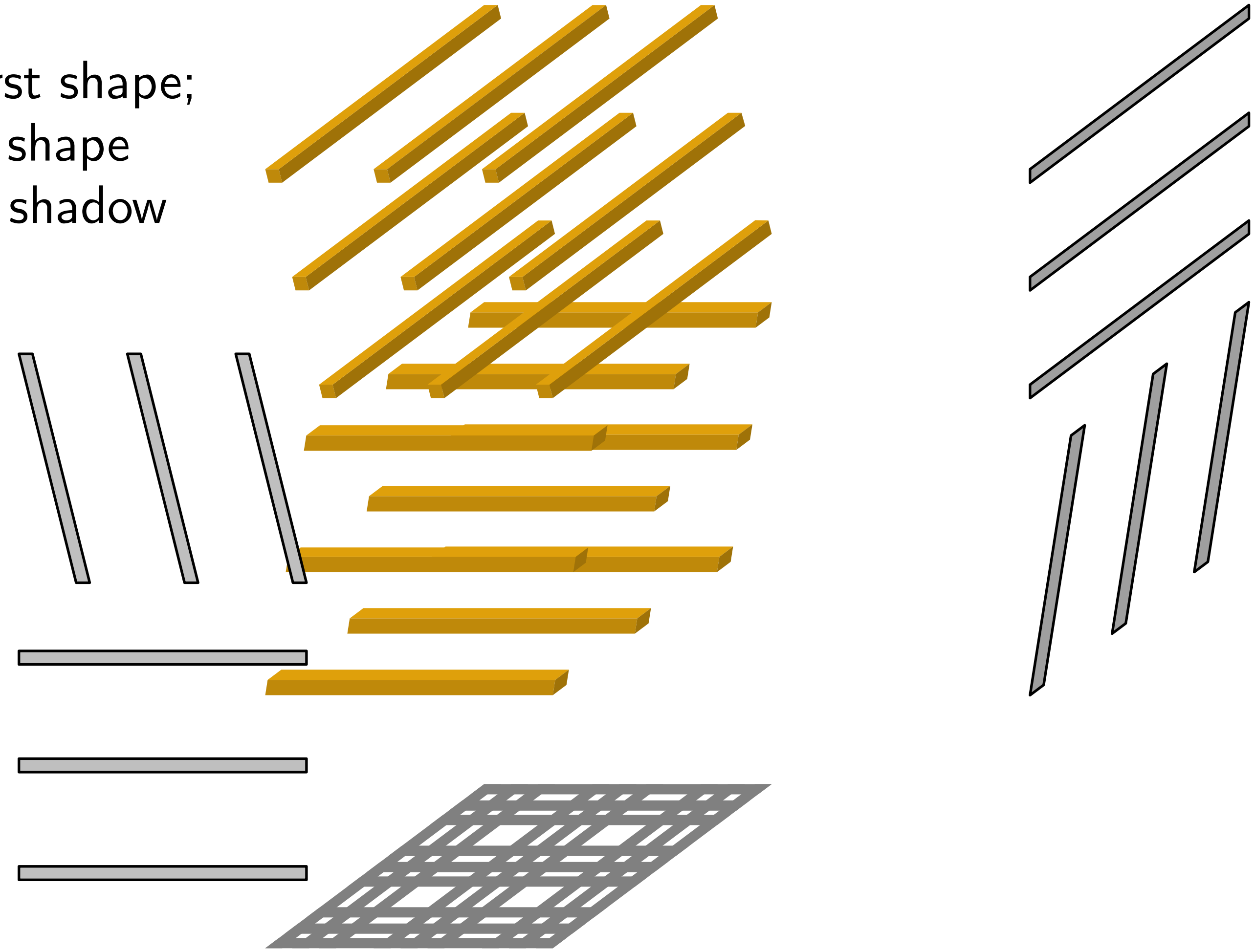

- For rectilinear shapes, complexity is only $\Theta(n^2)$ in the worst case.

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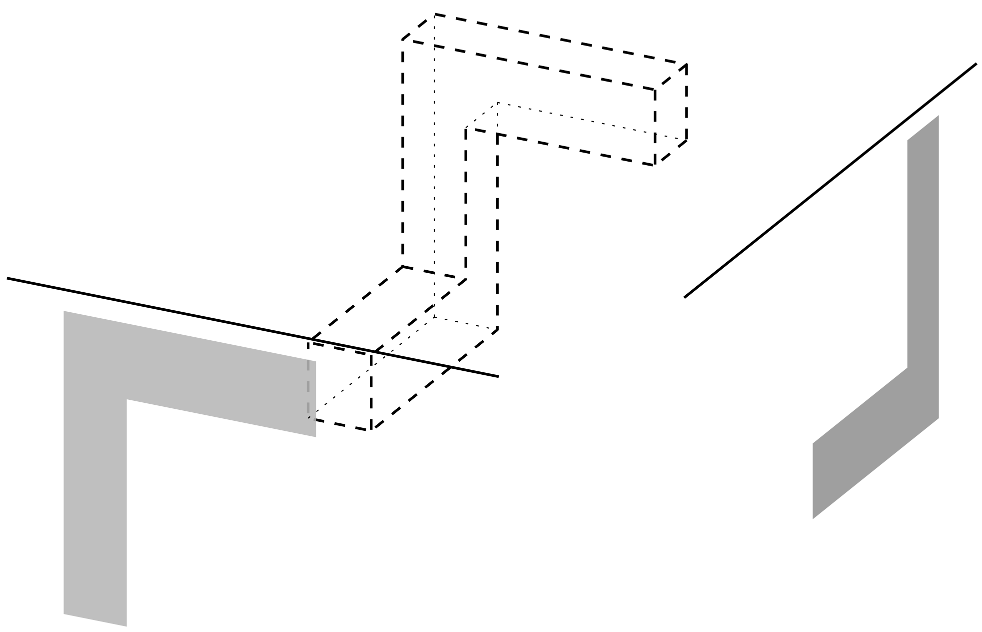

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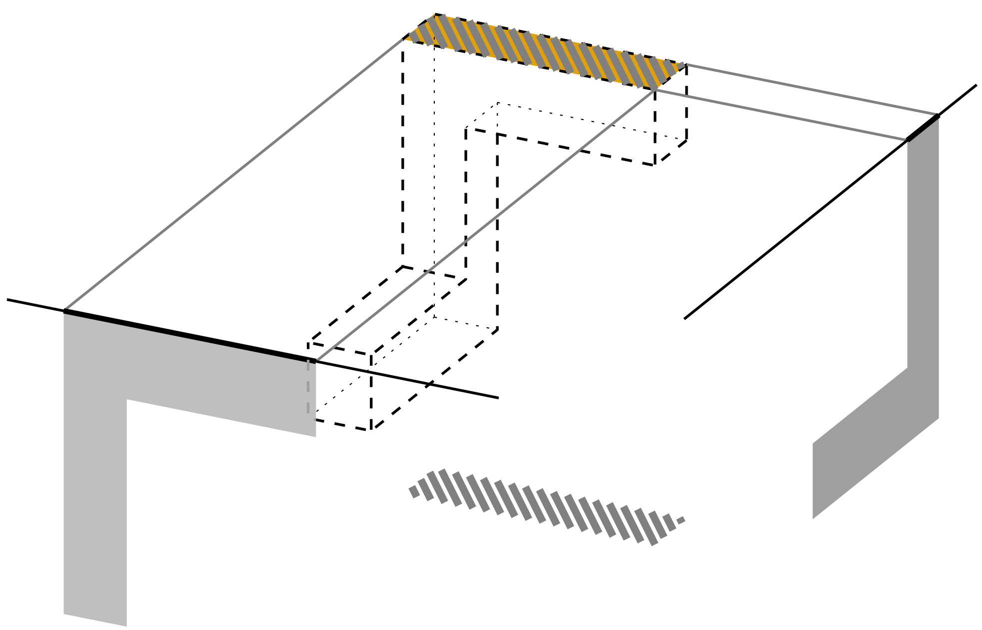

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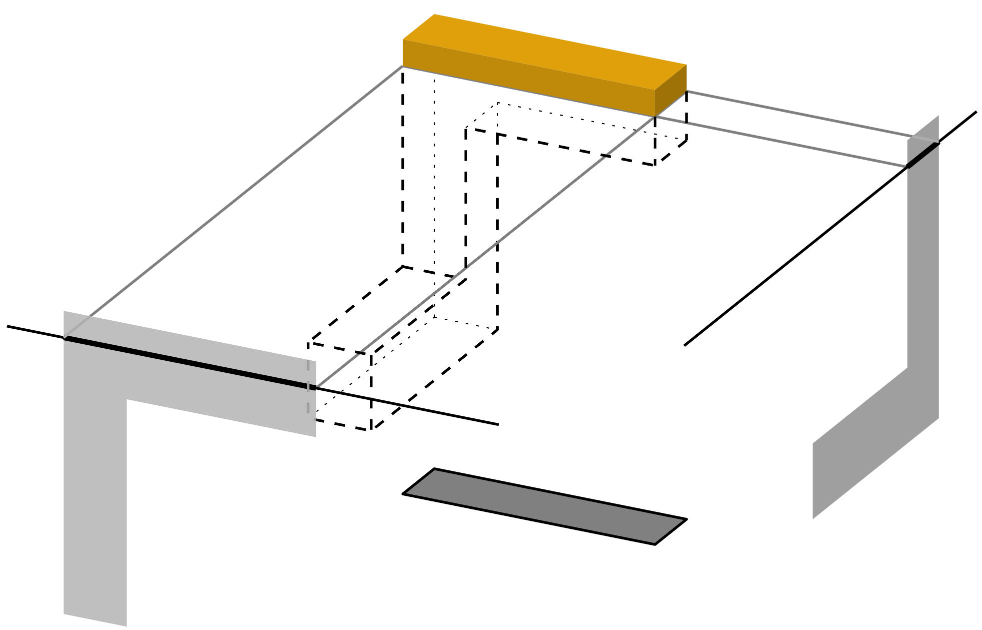

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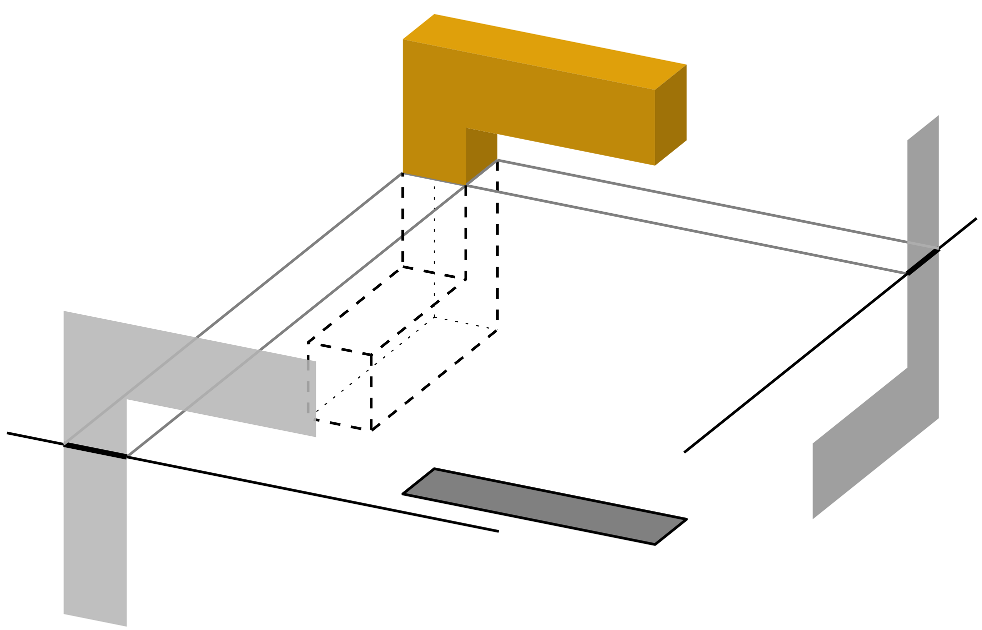

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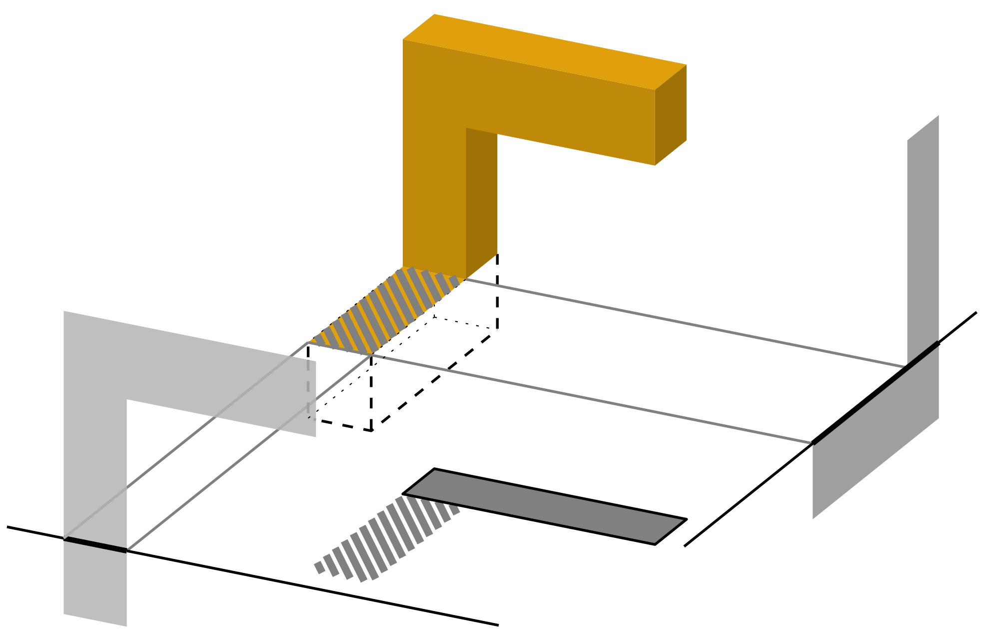

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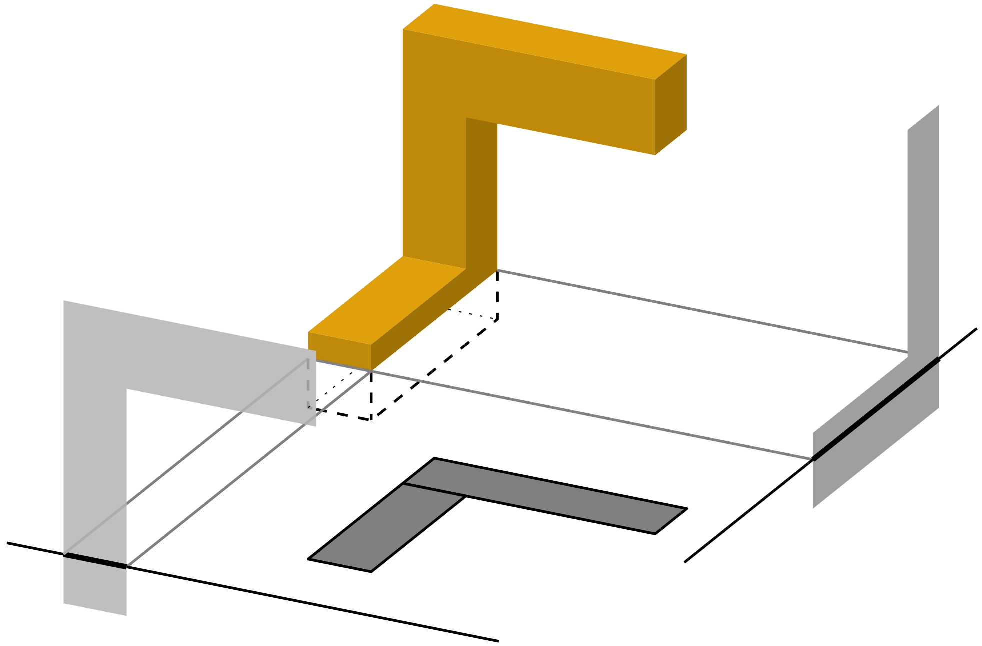

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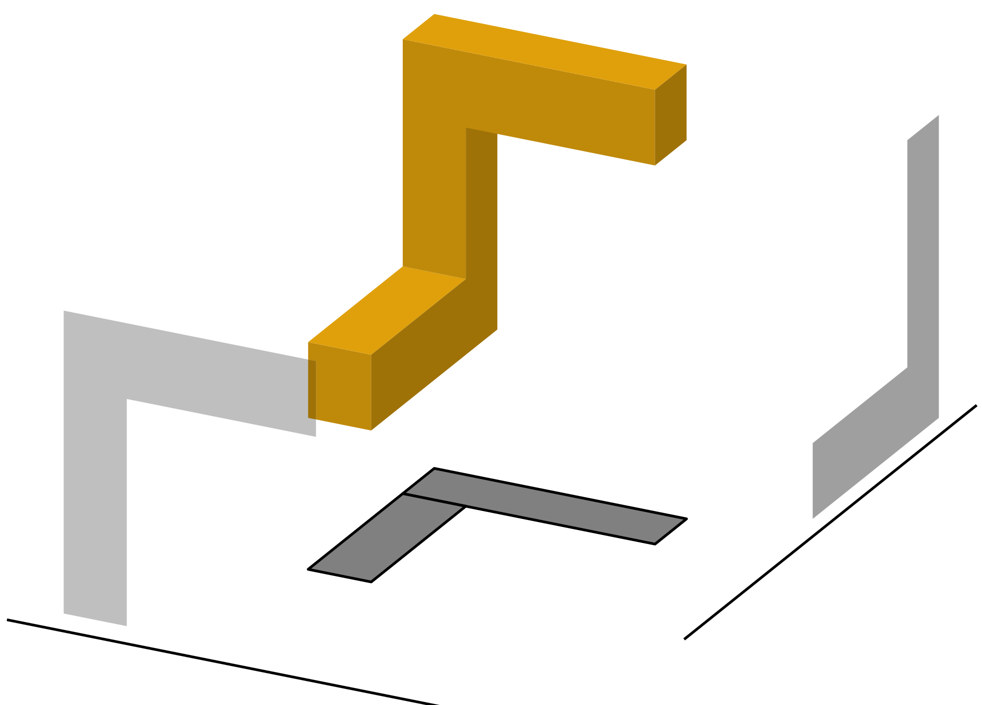

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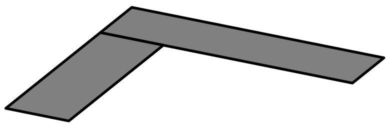
- Take union of rectangle set (size: $O(n^2)$), and compare shadow to shape.

  How to do this efficiently?



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  - Sweep-line algorithm.
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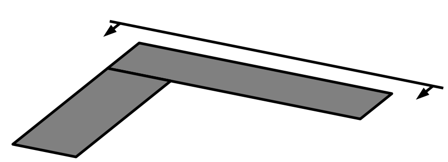

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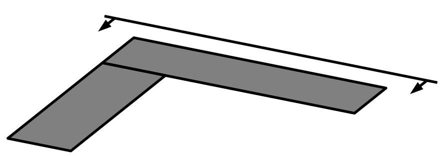

- Algorithm runs in $O(n^2 \log n)$ time.

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- Running time: $O((n^2 + k) \log n)$

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  3D Nef algorithm of Hachenberger et al. works for general polygons;
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- We know connected trip-lets have worst-case complexity $\Omega(n^2)$ and $O(n^3)$.

  Does there exist any connected trip-let with $\omega(n^2)$ vertices?

- Our algorithms are almost worst-case optimal for actually constructing the objects and their shadows;

  What about non-constructive algorithms?

  That is, can an algorithm be faster than $O(n^2)$ in the worst-case?